

# **A STUDY ON REGULAR MATRICES OVER AN INCLINE**

Thesis submitted to the Karpagam University in partial fulfillment of the  
requirements for the award of the degree of

**DOCTOR OF PHILOSOPHY**  
IN MATHEMATICS

*Submitted by*

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**JANUARY 2011**

## DECLARATION

I **P.SHAKILA BANU** hereby declare that the thesis entitled “**A STUDY ON REGULAR MATRICES OVER AN INCLINE**” submitted to the Karpagam University, in partial fulfillment of the requirements for the award of the Degree of Doctor of Philosophy in Mathematics is a record of bonafide and independent research work done by me during the period from January 2009 to December 2010 under the supervision and guidance of **Dr.(Mrs.)AR.MEENAKSHI**, Department of Mathematics and it has not formed the basis for the award of any Degree / Diploma / Associateship / Fellowship or similar title to any candidate of any other University.

**Signature of the Candidate**

## **CERTIFICATE**

This is to certify that the thesis entitled “**A STUDY ON REGULAR MATRICES OVER AN INCLINE**” submitted to the Karpagam University, in partial fulfillment of the requirement for the award of the Degree of Doctor of Philosophy in Mathematics is a record of bonafide research work done by **Mrs. P.SHAKILA BANU** during the period from January 2009 to December 2010 in the Department of Mathematics at Karpagam University under my supervision and guidance and the thesis has not formed the basis for the award of any Degree / Diploma / Associateship / Fellowship or similar title to any candidate of any other University.

**Countersigned**

**Signature of the Guide**

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